ELECTRIC QUADRUPOLE MOMENTS OF THE DECUPLET AND THE STRANGENESS CONTENT OF THE PROTON.

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ABSTRACT

In the SU3 Skyrme model the electric quadrupole moments of $\frac{3}{2}^+$ baryons show a strong sensitivity with respect to flavor distortions in baryon wavefunctions. SU3 symmetric wavefunctions lead to quadrupole moments proportional to the charge of the baryon whereas for strongly broken flavor symmetry a proportionality to baryonic isospin emerges. Since the flavor distortions in the wavefunctions also determine the strangeness content of the proton the Skyrme model provides a link between both quantities.

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This short note is concerned with the electric quadrupole moments $\langle B|\hat{Q}_{E2}|B\rangle$ of decuplet baryons B where recent quenched lattice gauge calculations[1] for the non-strange members of the decuplet find them to be proportional to their charge $\langle B|\hat{Q}_{E2}|B\rangle \sim \langle B|\hat{Q}|B\rangle = e\langle B|\frac{1}{2}\hat{Y} + \hat{I}_3|B\rangle$. These calculations are in slight contradiction with another recent prediction[2] from chiral perturbation theory where the quadrupole moments are closer to a proportionality to baryonic isospin alone, a pattern that also follows from the $SU2$-Skyrme model[1].

In the Skyrme model[3, 4] baryons are described as a hedgehog configuration $U_H = e^{i\tau\cdot\hat{r}\chi(r)}$ performing time dependent flavor rotations $A(t)$:

$$U(\mathbf{r}, t) = A(t)U_H(\mathbf{r})A^\dagger(t), \quad A^\dagger\dot{A} = -\frac{i}{2}\Omega_b\lambda_b. \tag{1}$$

For such a rotating hedgehog minimal substitution of the potential a^0 for a static electric field $\mathbf{E} = -\nabla a^0(\mathbf{r})$ into the Skyrme lagrangian simply adds the potential everywhere to the rotational velocities[5]

$$\dot{U} \to A\left\{[A^\dagger\dot{A}, U_H] + ia^0[A^\dagger QA, U_H]\right\}A^\dagger = -\frac{i}{2}\left(\Omega_b - ea^0D_{eb}(A)\right)A[\lambda_b, U_H]A^\dagger \tag{2}$$

We use the standard definitions for the D-functions in the regular representation, $D_{ab}(A) = \frac{1}{2}\operatorname{tr}\lambda_a A\lambda_b A^\dagger$, and the abbreviation that the index e stands for the linear combination of flavors entering into the charge operator, ($D_{ea} = D_{3a}$ for $SU2$ and $D_{ea} = D_{3a} + \frac{1}{\sqrt{3}}D_{8a}$ for $SU(3)$ e.g.).

Rotational velocities in the Skyme model lagrangian occur in two places: (i) quadratically in the rotational kinetic energy

$$T_{rot}[U] \to \frac{1}{2}\int d^3r\,\Theta_{ab}(\mathbf{r})(\Omega_a - ea^0D_{ea})(\Omega_b - ea^0D_{eb}). \tag{3}$$

There the density for the moments of inertia $\Theta_{ab}(\mathbf{r})$ is spherical outside $SU2$-subspace. Thus only the pionic inertia $a, b \in \{1, 2, 3\}$ have non-spherical components. (ii) Rotational velocities arise linearly from the anomalous part of the action which couples the winding number density to static electric potentials. But winding number density, again, is spherical for hedgehogs. In contrast to the mean square radius, i.e. the monopole moment, which receives contributions from the pionic inertia, the kaonic inertia $a, b \in \{4, \cdots, 7\}$, the Wess-Zumino term and further non-minimal photocouplings[6]

$$\Delta\mathcal{L} = il_9\,a_{\mu\nu}\,\operatorname{tr}Q(\nabla^\mu U\nabla^\nu U^\dagger + \nabla^\mu U^\dagger\nabla^\nu U) \tag{4}$$

the quadrupole moment originates from the rotational motion in $SU2$-subspace alone, giving:

$$\langle B|\hat{Q}_{E2}|B\rangle = \frac{e}{5}r_\pi^2\langle B|\,3D_{e3}J_3 - \sum_{i=1}^{3}D_{ei}J_i\,|B\rangle \tag{5}$$

where

$$r_\pi^2 = \frac{\int d^3r\,r^2\Theta_\pi(r)}{\int d^3r\Theta_\pi(r)}\,, \qquad \Theta_\pi(r) = \frac{1}{3}\sum_{a=1}^{3}\Theta_{aa}(\boldsymbol{r})\,, \tag{6}$$

is the pionic contribution to - and different from - the isovectorial r.m.s. radius of the nucleon, (see ref.[7]).

The matrix elements of the quadrupole operator in eq(5), involve the spin operator $J_a = -\int d^3r\Theta_\pi\Omega_a$ of the baryon and must be calculated using the eigenfunctions diagonalizing the rotational hamiltonian plus the SU3-symmetry breaking terms, a method pioneered by Yabu and Ando[8] and refined to a "slow rotator approximation" in ref.[9, 7]. Two limiting cases, however, may be given explicitly, the $SU3$-symmetric case and the limit where $SU3$-symmetry breaking becomes infinite:

$$\langle B|\hat{Q}_{E2}|B\rangle = \begin{cases} -\frac{1}{10}\,e\,r_\pi^2\langle B|\frac{1}{2}\hat{Y} + \hat{I}_3|B\rangle & SU3-\text{symmetric case,} \\[2ex] -\frac{4}{25}\,e\,r_\pi^2\alpha_B\langle B|\hat{I}_3|B\rangle & \text{strong symmetry breaking limit,} \\ & \text{with } \alpha_B = \{1, \frac{5}{4}, \frac{5}{3}, -\} \\ & \text{for } B = \{\Delta, \Sigma^*, \Xi^*, \Omega\}. \end{cases} \tag{7}$$

The quadrupole moments of the Δ's in the strong symmetry breaking limit coincide with the expressions given in the $SU2$-Skyrme model[1]. One can see that, as the $SU3$-symmetry breaking terms increase the admixture of higher representations to the pure decuplet wave functions, contributions proportional to the hypercharge of the baryon must become suppressed. On the other hand, a stronger mixing of higher representations also leads to a reduction of the strangeness content, $\langle\bar{s}s\rangle_B$, in the baryons. Thus we can correlate the quadrupole moments with the strangeness content for e.g. the proton. In the Skyrme model the latter ranges from $\langle\bar{s}s\rangle_p = 0$ in the strong symmetry breaking limit to $\langle\bar{s}s\rangle_p = \frac{7}{30}$ for $SU3$-symmetry[10]. The figure compiles this information for the baryon decuplet and one can see that for the case of the Δ's a proportionality with respect to isospin is already reached for empirical symmetry breaking fixed by physical meson masses and decay constants in the lagrangian (vertical line at $\langle\bar{s}s\rangle_p = .16$). With decreasing hypercharge the quadrupole moments, however, tend to the $SU3$-symmetric limit.
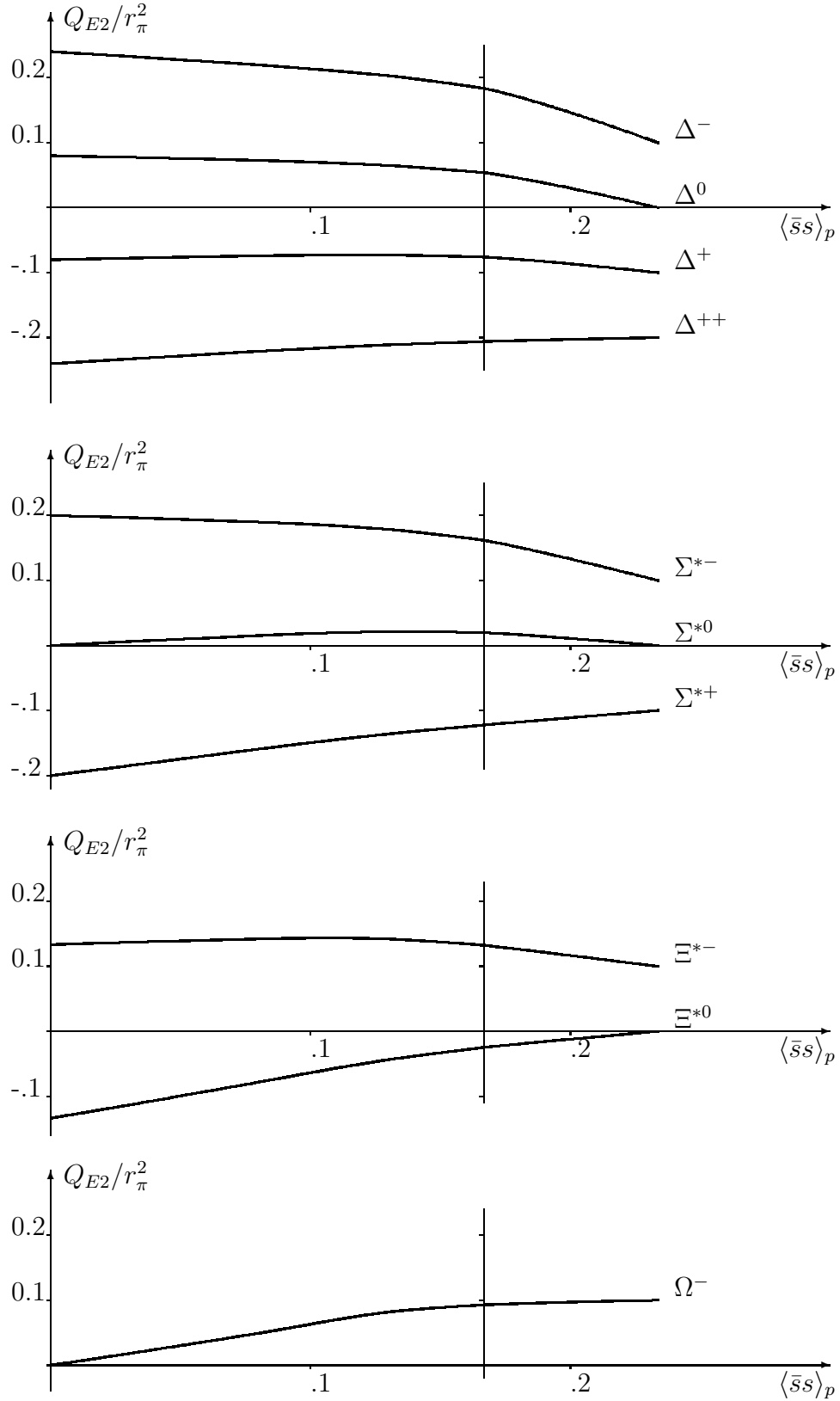
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Figure 1: Quadrupole moments versus the strangeness content of the proton. The functions have been obtained in rigid rotator approximation, ref.[9], with the parameters quoted there. The strangeness content is varied through a change of the kaon mass m_K. The vertical line indicates the position where $m_K = 495\text{MeV}$.

In plotting the quadrupole moments versus the strangeness content of the proton we hope to have removed some model dependence out of our statements, which are contained, for example, in assumptions on the exact form of higher order terms in the effective lagrangian. Nevertheless, we would like to terminate this short note by presenting (model dependent) numbers for these moments in table 1. as they follow from the slow rotator approach (case SK4 in ref.[7]) to the $SU3$-rotational motion of the soliton:

B	SRA	CPT	B	SRA	CPT
Δ^{++}	-0.87	-0.8$\pm$ 0.5	Σ^{*+}	-0.42	-0.7$\pm$ 0.3
Δ^{+}	-0.31	-0.3$\pm$ 0.2	Σ^{*0}	+0.05	-0.13$\pm$0.07
Δ^{0}	+0.24	+0.12$\pm$ 0.05	Σ^{*-}	+0.52	+0.4 $\pm$0.2
Δ^{-}	+0.80	+0.6$\pm$ 0.3			
			Ξ^{*0}	-0.07	-0.35$\pm$ 0.2
Ω^{-}	+0.24	+0.09$\pm$ 0.05	Ξ^{*-}	+0.35	+0.2$\pm$ 0.1

Table 1. Quadrupole moments of the baryon decuplet in units 10^{-1}e $\cdot$ fm^2 in slow rotator approximation (SRA) compared to chiral perturbation theory[2] (CPT).

For the non-strange Δ's chiral perturbation theory and the Skyrme model, both approaches, find a pattern proportional to isospin which in the case of chiral perturbation theory persists also for the strange members of the decuplet, whereas the Skyrme model moves closer to charge proportionality. As far as the magnitude of the moments is concerned, there seems to be mutual agreement, but at least in the case of the Skyrme model calculation there is a caveat : the isovector radius $\langle r^2 \rangle_V = \langle r^2 \rangle_p - \langle r^2 \rangle_n$ comes out too small ($\langle r^2 \rangle_V = .49$fm^2 for case SK4 in ref.[7]). Thus, the ratio of quadrupole moment to isovector radius is rather high in the Skyrme model as was correctly noticed in ref.[1]. In the slow rotator approximation it is roughly a factor of two higher than the corresponding ratio of the lattice gauge calculation[1].

In conclusion we have shown that three different approaches to the electric quadrupole moments of decuplet baryons: a quenched lattice gauge calculation, chiral perturbation theory and the Skyrme model, lead to three slightly different predictions which apparently differ in the amount of $SU3$ symmetry breaking in the decuplet states: the lattice gauge results are closest to $SU3$ symmetry whereas chiral pertur-

bation theory is closest to the strong symmetry breaking limit. The $SU3$ Skyrme model, finally, allows a smooth interpolation between the two limits as a function of the strangeness content of the proton and predicts a pattern intermediate between those mentioned, if empirical $SU3$ symmetry breaking is employed.

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